A METHOD TO DETERMINE THE PACKING AND GAS TEMPERATURES IN A ROTATING REGENERATOR WITH DISPERSE PACKING

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Equations are derived to determine the gas, air, and packing temperatures for any gas and air periods of regenerator operation, making possible regenerator design and complete analysis of apparatus operation.

Determination of packing and gas (air) temperatures for the beginning of apparatus operation was treated in [1]. The purpose of this paper is to determine the temperature field of the packing and gases (air) for apparatus operating in the steady-state regime.

Let us determine the packing temperature during the first air period following the first gas period examined in [1]. In analogy with (6) and (7) from [1] for any air period we can obtain the following equations:

$$\frac{\partial t^a}{\partial \lambda} = \vartheta^a - t^a, \tag{1}$$

$$-\frac{\partial t^a}{\partial \delta} = \vartheta^a - t^a. \tag{2}$$

Let us introduce the notation

$$\Theta^a(\lambda,\ \delta) = \frac{\vartheta^a - t_1^a}{t_1^g - t_1^a}. \tag{3}$$

The notations have been taken from [1].

With consideration of (3) from (1) and (2) we obtain the differential equation for the packing temperature during the air period

$$\frac{\partial^2 \Theta^a}{\partial \lambda \partial \delta} + \frac{\partial \Theta^a}{\partial \lambda} + \frac{\partial \Theta^a}{\partial \delta} = 0. \tag{4}$$

Let us formulate the boundary conditions for (4). In analogy with (10) and (11) from [1], according to [2], we have

$$\vartheta(0,\ \delta) = t_1^a + (\vartheta_1^a - t_1^a)\ \exp\ (-\delta) = t_1^a + \Delta t_1^a \exp\ (-\delta)$$

or with consideration of (3) we obtain

$$\Theta_1^a(0,\ \delta) = \frac{\Delta t_1^a \exp\ (-\delta)}{t_1^g - t_1^a}. \tag{5}$$

The initial condition

$$\vartheta^a(\lambda,\ 0) = \vartheta^g(\lambda,\ \delta_{fv}^g) \tag{6}$$

or from (3) and (8) according to [1] with consideration of (6) we obtain

$$\Theta_1^a(\lambda, 0) = \frac{t_1^g - (t_1^g - \vartheta_1)\ \Theta_1^g(\lambda,\ \delta_{fv}^g) - t_1^a}{t_1^g - t_1^a} = 1 - \Theta_1^g(\lambda,\ \delta_{1fv}^g). \tag{7}$$

Here we assume $t_1^a = \vartheta_1$ (the packing temperature prior to onset of regenerator operation is equal to the temperature of the ambient air).

Let us apply the Laplace transform of the variable λ to (4):

$$\frac{d}{d\delta}[s\bar{\Theta}^a(s,\delta) - \Theta^a(0,\delta)] + s\bar{\Theta}^a(s,\delta) - \Theta^a(0,\delta) + \frac{d\bar{\Theta}^a(s,\delta)}{d\delta} = 0. \tag{8}$$

Using the boundary condition (5), after some modifications, from (8) we obtain the following differential equation:

$$\frac{d\bar{\Theta}_1^a(s,\delta)}{\bar{\Theta}_1^a(s,\delta)} = -\frac{s}{1+s}d\delta,$$

whose solution is

$$\bar{\Theta}_1^a(s,\ \delta) = C(s)\ \exp\left(-\frac{s}{1+s}\delta\right). \tag{9}$$

We determine C(s) from (9) and the initial condition (7). In the transform region condition (7) assumes the form

$$\Theta_1^a(\lambda,\ 0) = 1 - \Theta_1^g(\lambda,\ \delta_{fv}^g) \doteqdot \frac{1}{s} - \frac{1}{s}\ \exp\left(-\frac{s}{1+s}\delta_{1fv}^g\right). \tag{10}$$

Substituting (10) into (9) for $\delta = 0$, we obtain

$$C(s) = \frac{1}{s} - \frac{1}{s}\ \exp\left(-\frac{s}{1+s}\delta_{1fv}^g\right). \tag{11}$$

Equation (9) then assumes the form

$$\bar{\Theta}_1^a(s,\delta) = \frac{1}{s}\ \exp\left(-\frac{s}{1+s}\delta\right) - \frac{1}{s}\ \exp\left[-\frac{s}{1+s}(\delta+\delta_{1fv}^g)\right]. \tag{12}$$

According to (28) from [1], the original of (12) is

$$\Theta_1^a(\lambda,\delta) = \exp(-\delta) + [\exp(-\lambda-\delta)] \times \sum_{k=1}^{\infty}\frac{\delta^k}{k!}\left(\exp\lambda - \sum_{n=0}^{k-1}\frac{\lambda^n}{n!}\right) - \exp(-\delta-\delta_{1fv}^g) - [\exp(-\lambda-\delta-\delta_{1fv}^g)] \times$$

$$\times \sum_{k=1}^{\infty} \frac{(\delta+\delta_{fv}^{g})^k}{k!} \times \left(\exp\lambda - \sum_{n=0}^{k-1}\frac{\lambda^n}{n!}\right).$$

After a number of modifications we finally obtain an expression for the packing temperature during the first air period

$$\Theta_1^a(\lambda,\delta) = \exp(-\delta) - \exp(-\delta-\delta_{1fv}^g) + [\exp(-\lambda-\delta)]\sum_{k=1}^{\infty}\frac{1}{k!}[\delta^k - (\delta+\delta_{1fv}^g)^k \exp(-\delta_{1fv}^g)]\left(\exp\lambda - \sum_{n=0}^{k-1}\frac{\lambda^n}{n!}\right). \qquad (13)$$

The equation to determine the air temperature during this period is derived, with consideration of (3), by substituting (13) into (2):

$$T_1^a = [\exp(-\lambda-\delta)] \times \sum_{k=1}^{\infty}\frac{1}{(k-1)!}[\delta^{k-1} - (\delta+\delta_{1fv})^{k-1}\exp(-\delta_{1fv}^g)] \times \left(\exp\lambda - \sum_{n=0}^{k-1}\frac{\lambda^n}{n!}\right), \qquad (14)$$

where

$$T_1^a(\lambda,\delta) = \frac{t^a(\lambda,\delta)-\vartheta_1}{t_1^g-\vartheta_1}. \qquad (15)$$

Let us consider the following period of regenerator operation which sets in on transition of the packing from the first air period to the following gas period, and it is this period to which we will refer as the second gas period. Consequently, the initial temperature distribution for the packing in this period is expressed by (13) when $\delta = \delta_{1fv}^a$ in accordance with (1) from [1].

The boundary condition for this case is

$$\Theta_2^g(0,\delta) = \frac{(t_1^g-\vartheta_1^g)\exp(-\delta)}{t_1^g-t_1^a},$$

where

$$\Theta_2^a(\lambda,\delta) = \frac{t_1^g-\vartheta^g(\lambda,\delta)}{t_1^g-\vartheta_1}. \qquad (16)$$

Since the equation describing the process of packing heating during the period in question is of the same form as (9) from [1], in analogy with the previous calculations we can derive the following equation for the determination of the packing temperature in the second gas period:

$$\Theta_2^g(\lambda,\delta) = \exp(-\delta) - \exp(-\delta-\delta_{1fv}^a) + \exp(-\delta-\delta_{1fv}^a-\delta_{1fv}^g) + [\exp(-\lambda-\delta)] \times \sum_{k=1}^{\infty}\frac{1}{k!}[\delta^k - (\delta+\delta_{1fv}^a)^k\exp(-\delta_{1fv}^g) + (\delta+\delta_{1fv}^a+\delta_{1fv}^g)^k\exp(-\delta_{1fv}^a-\delta_{1fv}^g)] \times \left(\exp\lambda - \sum_{n=0}^{k-1}\frac{\lambda^n}{n!}\right) \qquad (17)$$

and for the gas temperature in this period:

$$T_2^g = \frac{t_1^g - t^g(\lambda,\delta)}{t_1^g-\vartheta_1} = [\exp(-\lambda-\delta)]\sum_{k=1}^{\infty}\frac{1}{k!}[(\delta+\delta_{1fv}^a)^{k-1} \times \exp(-\delta_{1fv}^a) + (\delta+\delta_{1fv}^a+\delta_{1fv}^g)^{k-1} \times \exp(-\delta_{1fv}^a-\delta_{1fv}^g)] \times \left[\exp\lambda - \sum_{n=0}^{k-1}\frac{\lambda^n}{n!}\right]. \qquad (18)$$

Proceeding as in the previous cases, we successively derive an equation to determine the packing temperature in the second air period

$$\Theta_2^a = \frac{\vartheta^a(\lambda,\delta)-\vartheta_1}{t_1^g-\vartheta_1} = \exp(-\delta) - \exp(-\delta-\delta_{2fv}^g) + \exp(-\delta-\delta_{2fv}^g-\delta_{1fv}^a) - \exp(-\delta-\delta_{2fv}^g-\delta_{1fv}^a-\delta_{1fv}^g) + [\exp(-\lambda-\delta)] \times \sum_{k=1}^{\infty}\frac{1}{k!}[\delta^k - (\delta+\delta_{2fv}^g)^k\exp(-\delta_{2fv}^g) + (\delta+\delta_{2fv}^g+\delta_{1fv}^a)^k\exp(-\delta_{2fv}^g-\delta_{1fv}^a) - (\delta+\delta_{2fv}^g+\delta_{1fv}^a+\delta_{1fv}^g)^k \times \exp(-\delta_{2fv}^g-\delta_{1fv}^a-\delta_{1fv}^g)] \times \left[\exp\lambda - \sum_{n=0}^{k-1}\frac{\lambda^n}{n!}\right] \qquad (19)$$

and the equation to determine the air temperature in the second air period

$$T_2^a = \frac{t^a(\lambda,\delta)-\vartheta_1}{t_1^g-\vartheta_1} = [\exp(-\lambda-\delta)]\sum_{k=1}^{\infty}\frac{1}{(k-1)!}[\delta^{k-1} - (\delta+\delta_{2fv}^g)^{k-1}\exp(-\delta_{2fv}^g) + (\delta+\delta_{2fv}^g+\delta_{1fv}^a)^{k-1} \times \exp(-\delta_{2fv}^g-\delta_{1fv}^a) - (\delta+\delta_{2fv}^g+\delta_{1fv}^a+\delta_{1fv}^g)^{k-1} \times \exp(-\delta_{2fv}^g-\delta_{1fv}^a-\delta_{1fv}^g)]\left[\exp\lambda - \sum_{n=0}^{k-1}\frac{\lambda^n}{n!}\right]. \qquad (20)$$

In complete analogy it is possible to derive an equation for the temperature of the packing and the gases in the subsequent gas and air period. However, from the derived (27) and (29) from [1], as well as from (13), (14), and (17)–(20) we can see the quantitative relationship governing the form of the functions to determine the temperatures of the packing and gases in

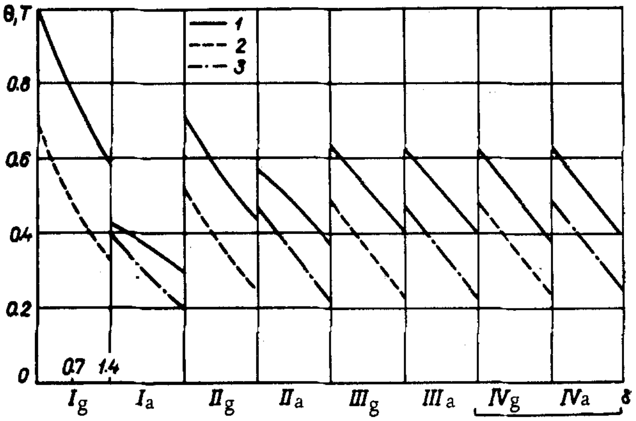

Fig. 1. Distribution of dimensionless packing temperature Θ and heat-transfer agent T (gases and air) in gas and air periods since the beginning of operation to steady-state process ($\lambda = 1.2$, $\delta = 0-1.4$, $\delta_{fv} = 1.4$): I–IV) period numbers IV_g and IV_a show the steady-state process); 1) packing; 2) gas; 3) air.

any of the subsequent periods. Thus, proceeding from (27) from [1] and from (17), for any gas period the equation for the determination of the passing temperature assumes the form

$$\begin{aligned}\Theta_m^g &= \exp(-\delta) - \exp(-\delta - \delta_{m-1}^a) + \exp(-\delta - \delta_{m-1}^a - \delta_{m-1}^g) - \\ &- \exp(-\delta - \delta_{m-1}^a - \delta_{m-1}^g - \delta_{m-2}^a) + \ldots + \exp(-\delta - \delta_{m-1}^a - \delta_{m-1}^g - \\ &- \delta_{m-2}^a - \ldots - \delta_1^a - \delta_1^g) + [\exp(-\lambda - \delta)] \times \\ &\times \sum_{k=1}^{\infty} \frac{1}{k!} [\delta^k - (\delta + \delta_{m-1}^a)^k \exp(-\delta_{m-1}^a) + \\ &+ (\delta + \delta_{m-1}^a + \delta_{m-1}^g)^k \exp(-\delta_{m-1}^a - \delta_{m-1}^g) - \\ &- (\delta + \delta_{m-1}^a + \delta_{m-1}^g + \delta_{m-2}^a)^k \times \\ &\times \exp(-\delta_{m-1}^a - \delta_{m-1}^g - \delta_{m-2}^a) + \ldots + \\ &+ (\delta + \delta_{m-1}^a + \delta_{m-1}^g + \delta_{m-2}^a + \ldots + \delta_1^a + \delta_1^g)^k \times \\ &\times \exp(-\delta_{m-1}^a - \delta_{m-1}^g - \delta_{m-2}^a - \ldots - \delta_1^a - \delta_1^g)] \times \\ &\times \left[\exp\lambda - \sum_{n=0}^{k-1} \frac{\lambda^n}{n!}\right] \qquad (21)\end{aligned}$$

(the subscript "fv" has been dropped here and beyond in all cases of dual subscripts) and the equation for the packing temperature in any air period, according to (13) and (19) assumes the form

$$\begin{aligned}\Theta_m^a &= \exp(-\delta) - \exp(-\delta - \delta_m^g) + \exp(-\delta - \delta_m^g - \delta_{m-1}^a) - \\ &- \exp(-\delta - \delta_m^g - \delta_{m-1}^a - \delta_{m-1}^g) + \ldots + \\ &+ \exp(-\delta - \delta_m^g - \delta_{m-1}^a - \delta_{m-1}^g - \ldots - \delta_1^a - \delta_1) + \\ &+ [\exp(-\lambda - \delta)] \times \\ &\times \sum_{k=1}^{\infty} \frac{1}{k!} [\delta^k - (\delta + \delta_m^g)^k \exp(-\delta_m^g) + \\ &+ (\delta + \delta_m^g + \delta_{m-1}^a)^k \exp(-\delta_m^g - \delta_{m-1}^a) - \\ &- (\delta + \delta_m^g + \delta_{m-1}^a + \delta_{m-1}^g)^k \times \\ &\times \exp(-\delta_m^g - \delta_{m-1}^a - \delta_{m-1}^g) + \ldots + \\ &+ (\delta + \delta_m^g + \delta_{m-1}^a + \delta_{m-1}^g + \ldots + \delta_1^a + \delta_1^g)^k \times \\ &\times \exp(-\delta_m^g - \delta_{m-1}^a - \delta_{m-1}^g - \ldots - \delta_1^a - \delta_1^g)] \times \\ &\times \left[\exp\lambda - \sum_{n=0}^{k-1} \frac{\lambda^n}{n!}\right]. \qquad (22)\end{aligned}$$

For the gas temperatures in any gas period in accordance with (29) from [1] and (18) we obtain the following expression:

$$\begin{aligned}T_m^g &= [\exp(-\lambda - \delta)] \sum_{k=1}^{\infty} \frac{1}{(k-1)!} \times \\ &\times [\delta^{k-1} - (\delta + \delta_{m-1}^a)^{k-1} \exp(-\delta_{m-1}^a) + \\ &+ (\delta + \delta_{m-1}^a + \delta_{m-1}^g)^{k-1} \exp(-\delta_{m-1}^a - \delta_{m-1}^g) - \\ &- (\delta + \delta_{m-1}^a + \delta_{m-1}^g + \delta_{m-2}^a)^{k-1} \times \\ &\times \exp(-\delta_{m-1}^a - \delta_{m-1}^g - \delta_{m-2}^a) + \ldots + \\ &+ (\delta + \delta_{m-1}^a + \delta_{m-1}^g + \delta_{m-2}^a + \ldots + \delta_1^a + \delta_1^g)^{k-1} \times \\ &\times \exp(-\delta_{m-1}^a - \delta_{m-1}^g - \delta_{m-2}^a - \ldots - \delta_1^a - \delta_1^g)] \times \\ &\times \left[\exp\lambda - \sum_{n=0}^{k-1} \frac{\lambda^n}{n!}\right] \qquad (23)\end{aligned}$$

and the equation for the air temperature in any air period according to (14) and (20) is given by

$$\begin{aligned}T_m^a &= [\exp(-\lambda - \delta)] \times \\ &\times \sum_{k=1}^{\infty} \frac{1}{(k-1)!} [\delta^{k-1} - (\delta + \delta_m^g)^{k-1} \exp(-\delta_m^g) + \\ &+ (\delta + \delta_m^g + \delta_{m-1}^a)^{k-1} \exp(-\delta_m^g - \delta_{m-1}^a) - \\ &- (\delta + \delta_m^g + \delta_{m-1}^a + \delta_{m-1}^g)^{k-1} \times \\ &\times \exp(-\delta_m^g - \delta_{m-1}^a - \delta_{m-1}^g) + \ldots + \\ &+ (\delta + \delta_m^g + \delta_{m-1}^a + \delta_{m-1}^g + \ldots + \delta_1^a + \delta_1^g)^{k-1} \times \\ &\times \exp(-\delta_m^g - \delta_{m-1}^a - \delta_{m-1}^g - \ldots - \delta_1^a - \delta_1^g)] \times \\ &\times \left[\exp\lambda - \sum_{n=0}^{k-1} \frac{\lambda^n}{n!}\right]. \qquad (24)\end{aligned}$$

In (21) and (24) the subscript m = 1, 2, 3, ... denotes the number of the corresponding period (after the first gas period we have the first air period, followed by the second gas period, etc.). If we assume that the heat-transfer coefficient for any gas and air periods, calculated with respect to average parameters, are equal to each other, and if we assume that the gas and air periods are identical in terms of duration, and if we neglect the fact that the temperature alters the heat capacity of the packing, it becomes possible to assume

$$\delta_m^{g} = \delta_{m-1}^{g} = \delta_{m-2}^{g} = \ldots = \delta_m^{a} = \delta_{m-1}^{a} = \delta_{m-2}^{a} = \delta_m. \quad (25)$$

With condition (25) for the instantaneous temperature of the packing in any gas period with the exception of the first and in any air period instead of (21) and (22) we obtain the following equation:

$$\Theta_m = [\exp(-\delta)] \sum_{p=1}^{2m-r} (-1)^p \exp(-\delta - p\,\delta_{fv}) + [\exp(-\lambda-\delta)] \times \sum_{k=1}^{\infty} \frac{1}{k!}\left[\delta^k + \sum_{p=1}^{2m-r} (-1)^p (\delta + p\,\delta_{fv})^k \times \exp(-p\,\delta_{fv})\right]\left[\exp\lambda - \sum_{n=0}^{k-1}\frac{\lambda^n}{n!}\right], \quad (26)$$

while for the instantaneous temperature of the gases in any gas period with the exception of the first and for the instantaneous air temperature in any air periods, instead of (23) and (24) we obtain

$$T_m = [\exp(-\lambda-\delta)] \times \sum_{k=1}^{\infty} \frac{1}{(k-1)!}\left[\delta^{k-1} + \sum_{p=1}^{2m-r} (-1)^p (\delta + p\,\delta_{fv})^{k-1} \times \exp(-p\,\delta_{fv})\right]\left[\exp\lambda - \sum_{n=0}^{k-1}\frac{\lambda^n}{n!}\right], \quad (27)$$

where r = 2 for the gas periods and r = 1 for the air periods.

The instantaneous temperatures of the packing and the gases in the first gas period can be calculated, respectively, from (27) and (29) in [9]. Θ_m and T_m are, respectively, determined from (3) and (15).

It should be noted that the series in (26) and (27) are absolutely converging.

The steady state sets in after a number of heating and cooling periods, theoretically repeating an infinite number of times. As we can see from (26) and (27), this state sets in at $\exp(-p\delta_{fvg}) \to 0$. Consequently, assuming accuracy of calculation, we can determine the number of heating and cooling periods subsequent to which the steady-state regime has, for all intents and purposes, set in. Thus we have $N/\delta_{fv} = 2m - r$, hence $m = 1/2\,(r + N/\delta_{fv})$, where N is determined on the basis of the selected level of accuracy.

Knowing the temperature of the gases and air at the regenerator outlet is of great practical importance. These temperatures may be calculated by averaging the corresponding local temperatures determined from (27).

Thus with $\lambda = \lambda_{fv}$, for the average gas and air temperature from (27) we have

$$T_{av} = \frac{1}{\delta_{fv}}\left\{\sum_{k=1}^{\infty}\frac{1}{(k-1)!}\left[(k-1)\int_0^{\delta_{fv}} \delta^{k-2} \times \exp(-\lambda_{fv} - \delta)\,d\delta - \delta_{fv}^{k-1}\exp(-\lambda_{fv} - \delta_{fv}) + \sum_{p=1}^{2m-r}(-1)^p\left[(p\,\delta_{fv})^{k-1}\exp(-p\,\delta_{fv}) - (\delta_{fv} + p\,\delta_{fv})^{k-1}\exp(-\delta_{fv} - p\,\delta_{fv}) + (k-1)\int_0^{\delta_{fv}}(\delta + p\,\delta_{fv})^{k-2}\exp(-\delta - p\,\delta_{fv})\,d\delta\right] \times \exp(-\lambda_{fv})\right]\right\}\left(\exp\lambda - \sum_{n=0}^{k-1}\frac{\lambda^n}{n!}\right). \quad (28)$$

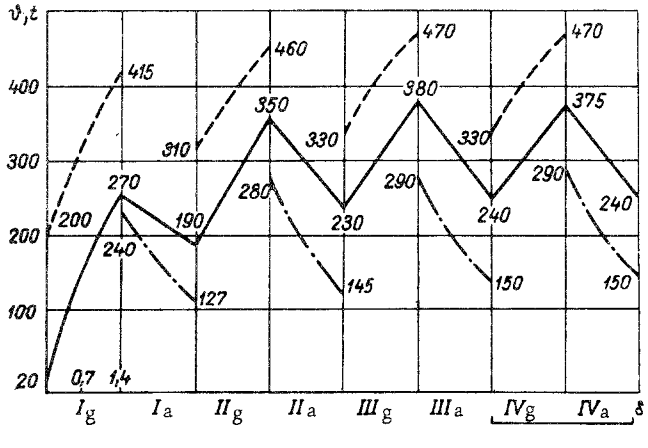

Fig. 2. Change in packing temperature ϑ and in the temperature t for the gases and air in the gas and air periods from the beginning of apparatus operation to the steady-state process. Notations are given in Fig. 1.

The integrals in (28) can be calculated in each specific case for a selected value of N.

The resulting equations make it possible completely to analyze regenerator operation, to determine the temperature of the gases and the heated air at the outlet from the regenerator, and they can also serve to determine the heat-transfer coefficients in the filtering of the gases through the dense packing layer for the case under consideration here.

NOTATION

Here t and ϑ are the instantaneous temperatures of the heat-transfer agent and the packing, respectively. Subscripts: a, air; g, gas; fv, finite value; 1, initial temperature or number of the first period.

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